Mathematical Challenges of the Euclid Spatial Project

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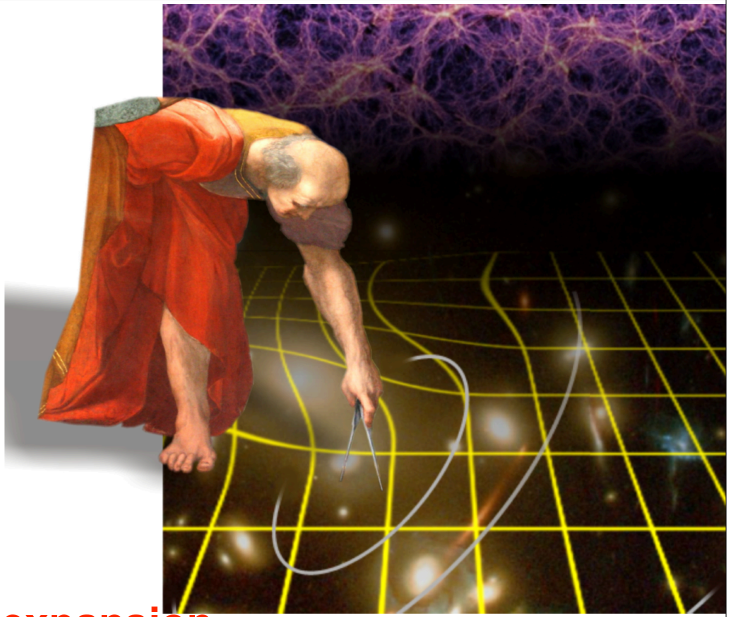
on behalf of **the Euclid Consortium & ESA**

Euclid



Euclid, ESA Cosmic Vision: launch in 2020:
- 800 members (450 researchers).
- 107 laboratories
- 13 countries

Euclid is the result from the fusion in 2008 of two missions:
- DUNE
- SPACE

Understand the origin of the Universe's accelerating expansion

→probe the properties and nature of *dark energy, dark matter, gravity* and distinguish their effects decisively

→by tracking their observational signatures on the
- geometry of the universe: **Weak Lensing** + Galaxy Clustering
- cosmic history of structure formation: **WL**, z-space distortion, clusters of galaxies

Gains in space:

Stable data: homogeneous data set over the whole sky

→**Systematics** are small, understood and controlled

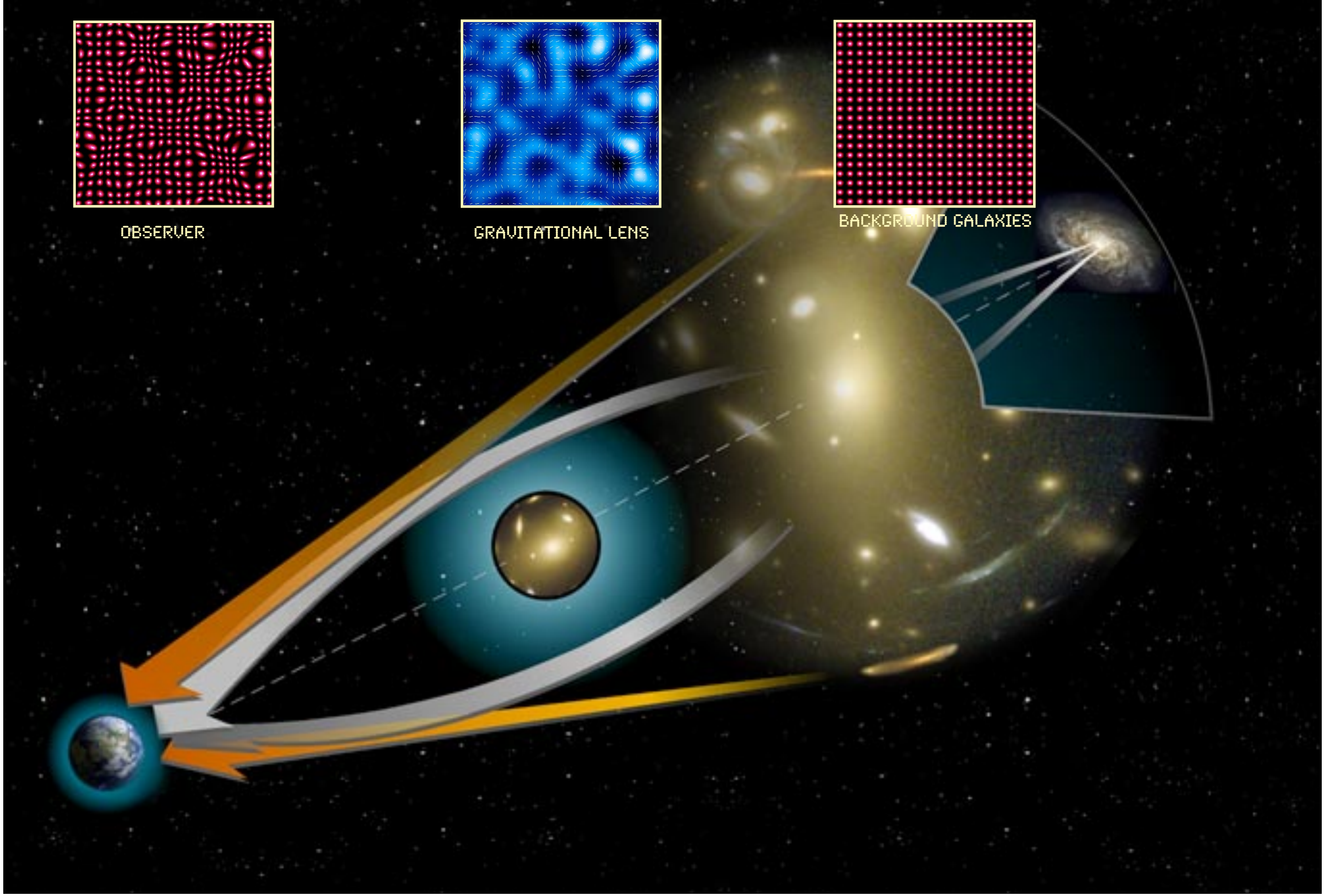
→Homogeneity : Selection function perfectly controlled

- Observe **15 000 deg2** during 7 years in optical and near infrared wavelength. (forme) dans le visible ET
- 1,2m Telescope, 4 bands
- Photometric redshift (distance) for **1 000 000 000 galaxies.**
- Spectroscopic IR measurement of **50 000 000 galaxies.**

==> 850 Gbits of data per day.

http://www.euclid-ec.org/

Weak Lensing



Tomographic Weak Lensing

THE ASTROPHYSICAL JOURNAL, 522:L21–L24, 1999 September 1

POWER SPECTRUM TOMOGRAPHY WITH WEAK LENSING

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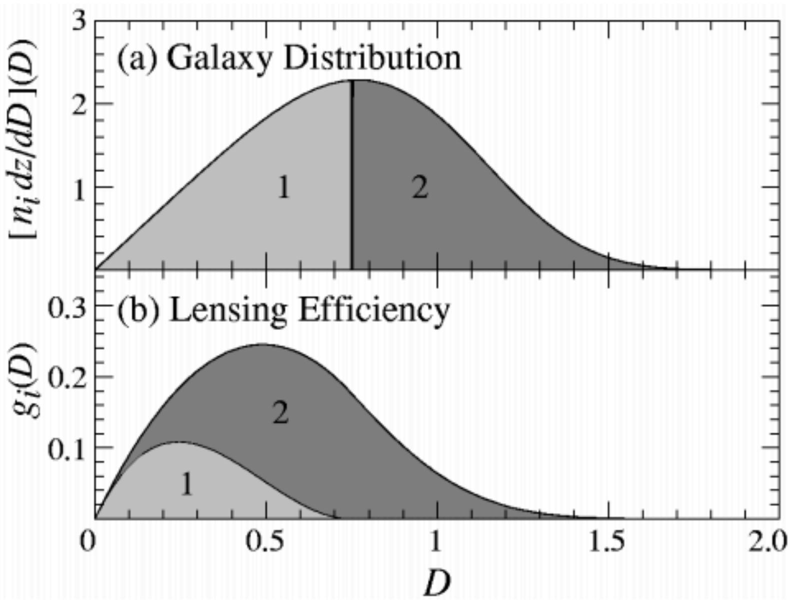
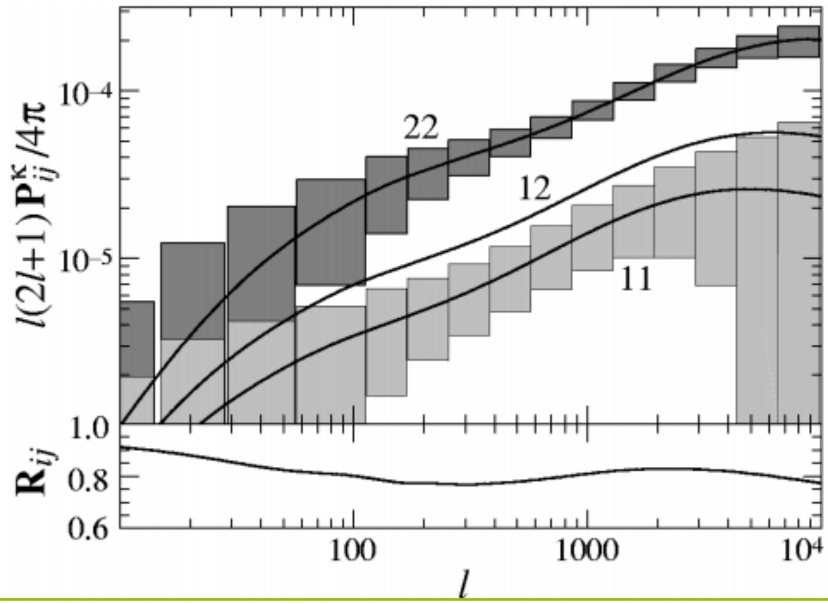
Received 1999 April 13; accepted 1999 June 30; published 1999 July 21





Euclid Red Book

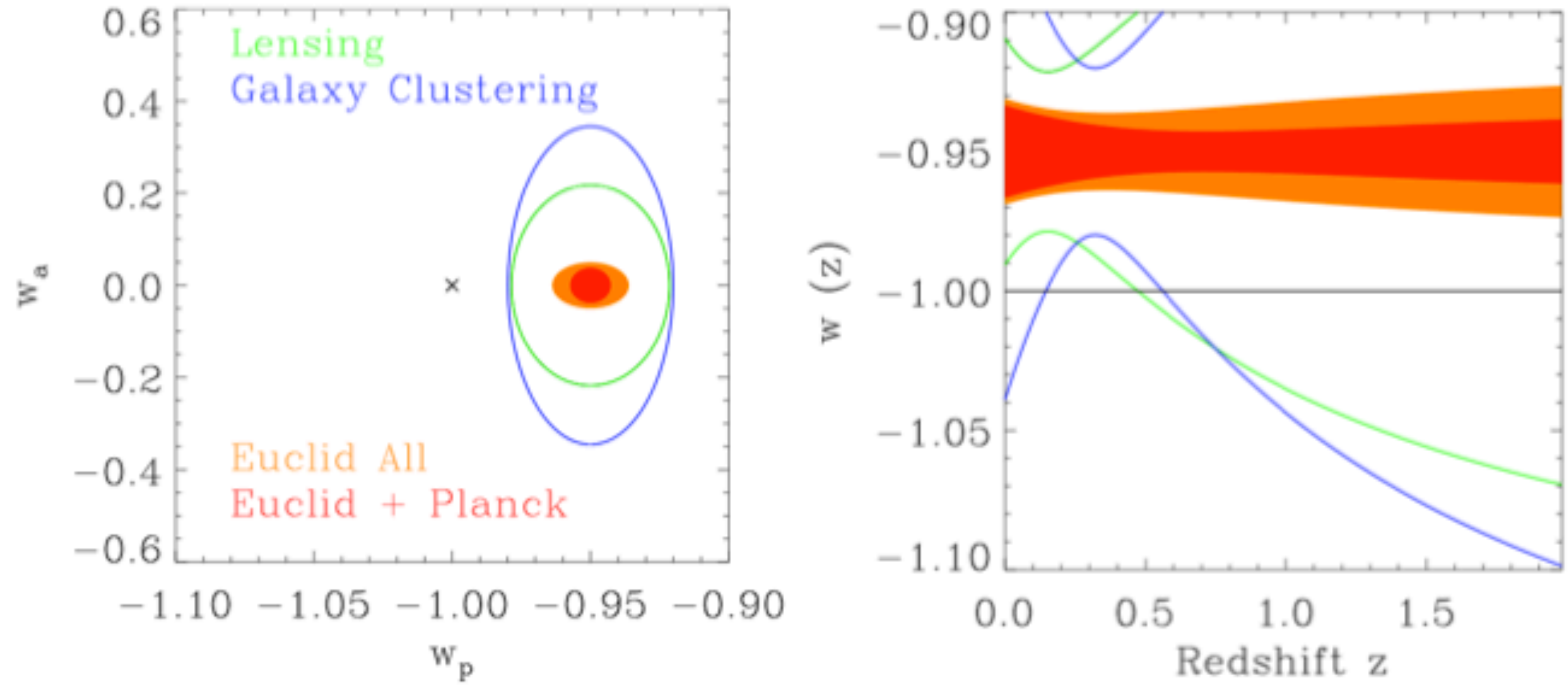
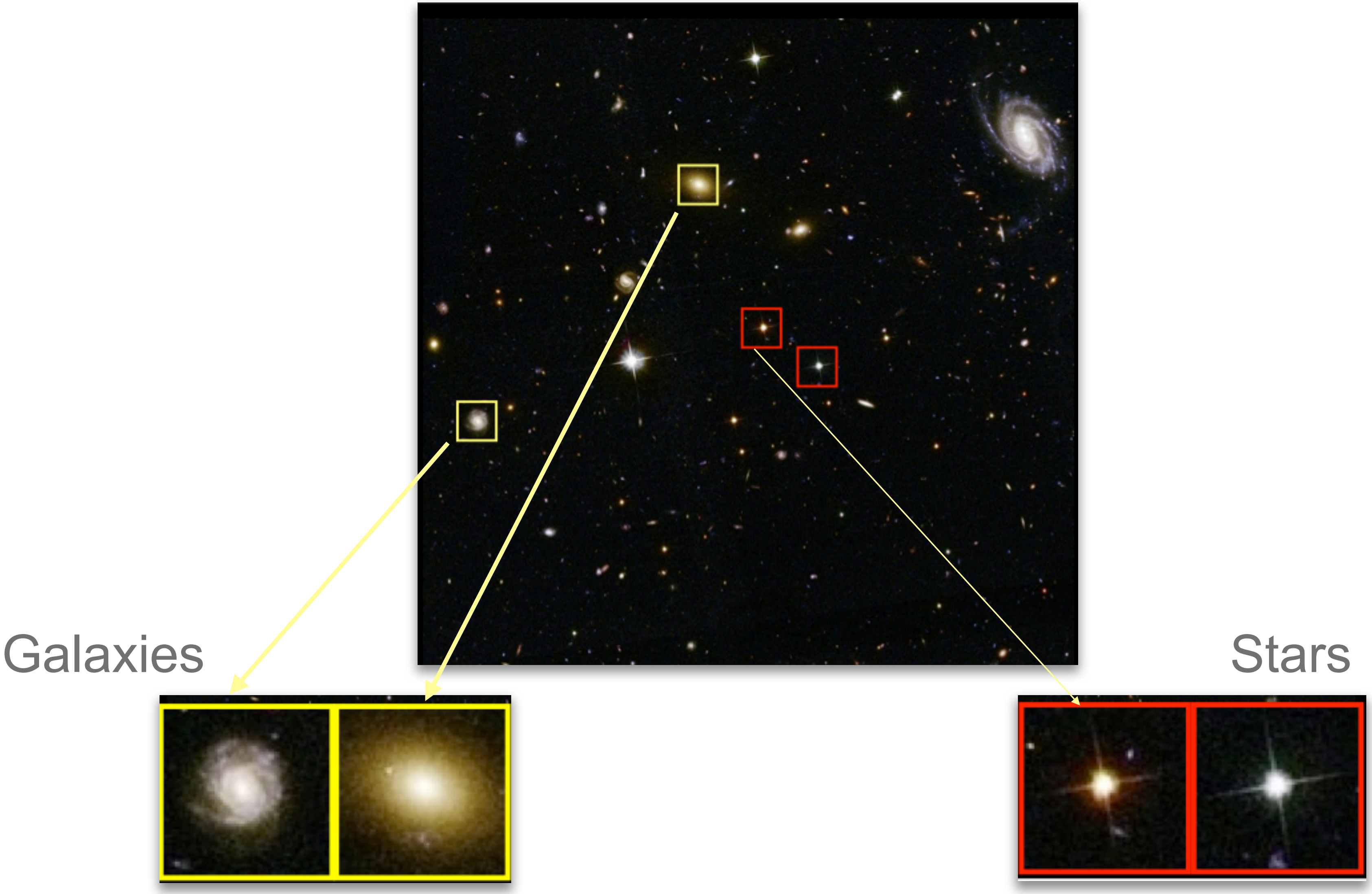


Figure 2.4: The expected constraints from Euclid in the dynamical dark energy parameter space. We show lensing only (green), galaxy clustering only (blue), all the Euclid probes (lensing+galaxy clustering+clusters+ISW; orange) and all Euclid with Planck CMB constraints (red). The cross shows a cosmological constant model. Left panel: the expected 68% confidence contours in the (w_p, w_a). Right panel: the 1σ constraints on the function w(z) parameterised by (w_p, w_a) as a function of redshift (green-lensing alone, blue-galaxy clustering alone, orange-all of the Euclid probes, red-Euclid combined with Planck).

Detection + Classification stars/galaxies

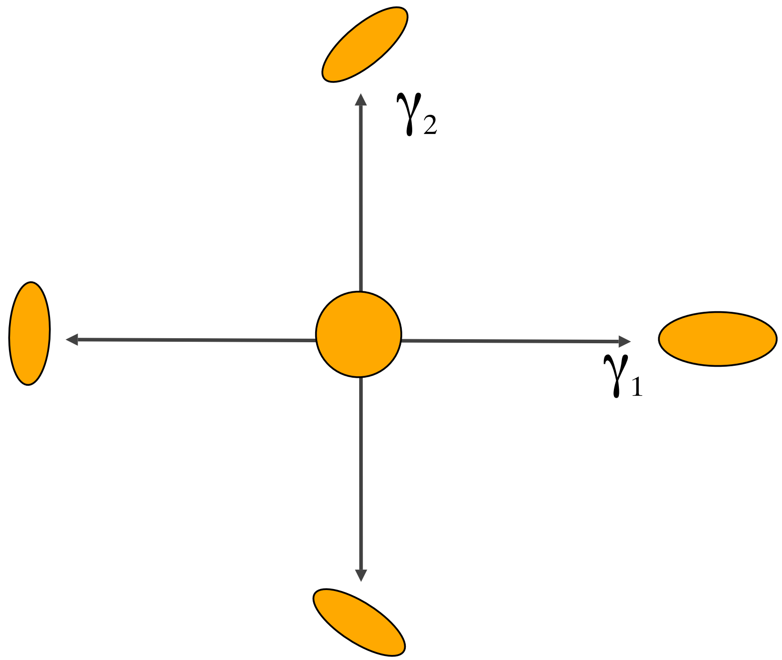


Shape Parameters

The shear map (γ_1, γ_2)

γ_1 = deformation along the x-axis,
and γ_2 at 45 degrees from it.

$$\gamma = \gamma_1 + i\gamma_2 = |\gamma| e^{2i\theta}$$



Where the modulus represents the amount of shear and the phase represents its direction.

$$\gamma_1 = \frac{M_{1,1} - M_{2,2}}{M_{1,1} + M_{2,2}}, \gamma_2 = \frac{2M_{1,2}}{M_{1,1} + M_{2,2}}, \quad M_{i,j} = \int \theta_i \theta_j S(\theta) w(\theta) d\theta^2$$

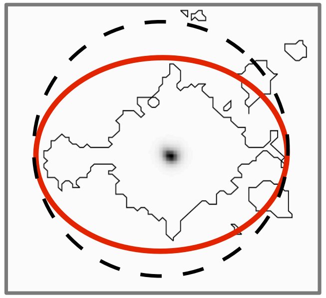
$M_{1,1} - M_{2,2}$ and $2M_{1,2}$ correspond respectively to the flattening along the x axis and the 45° axis. $M_{1,1} + M_{2,2}$ is related to the size.

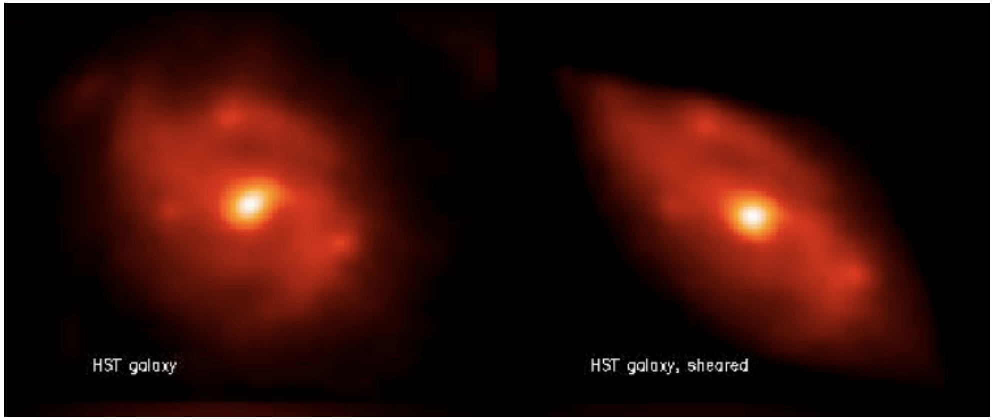
PB 1: We need accurate measurements from noisy data

Point Spead Function

Galaxies are convolved by an asymetric PSF

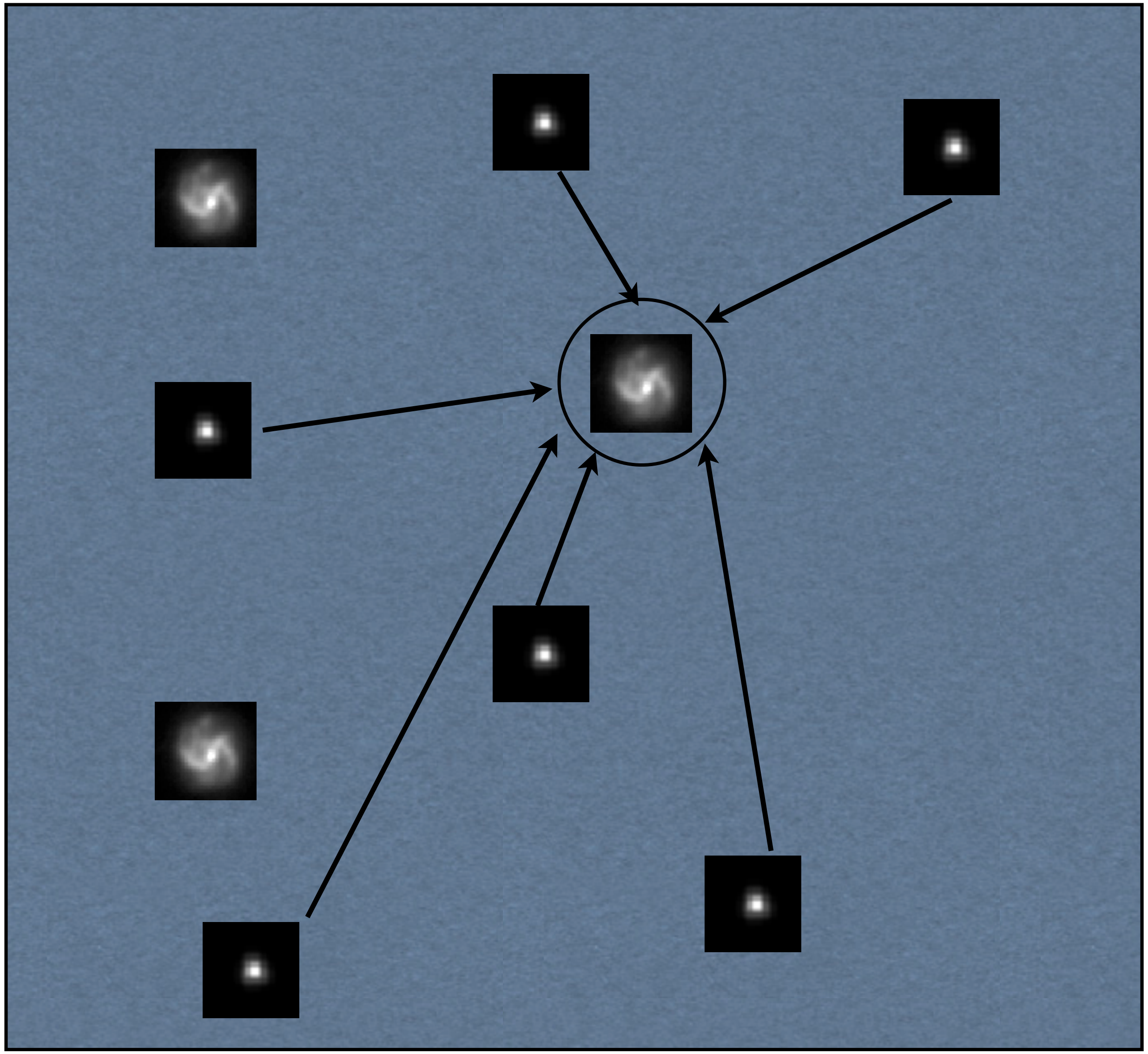
Convolution with an isotropic PSF circularises galaxies.
Convolution with an anisotropic PSF also changes their shapes... coherently!
Worst from ground (large PSF, with unpredictable spatial / temporal variation).





PB 2: Shape measurements must be deconvolved

Space Variant PSF



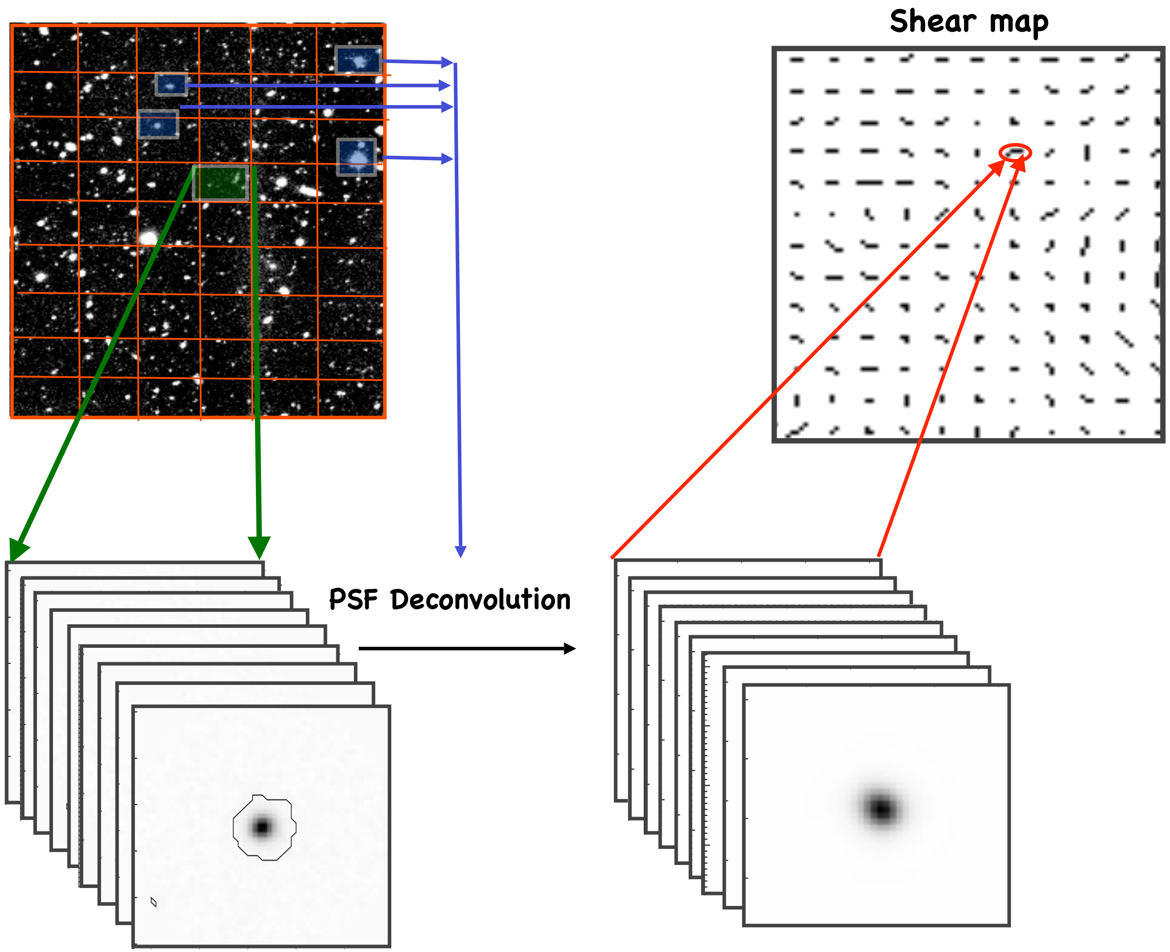
PB 3: We need to interpolate the PSF shape !

Intrinsic Ellipticities

✓ Galaxies have an intrinsic ellipticipty

PB 4: We need to correct the measurements from the intrinsic ellipticity

From shear measurements to shear map



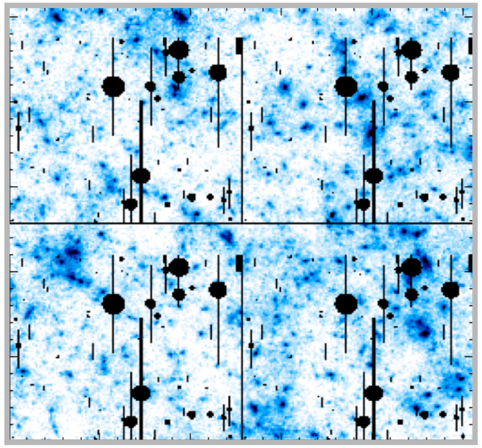
To Build a Shear Catalog
We need to solve a triple inverse problem !!!

1) Determine the PSF at any position from the measured PSF.

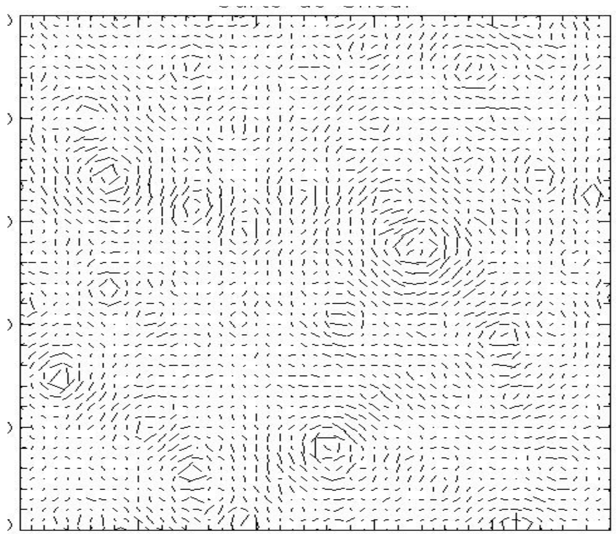
2) Measure the galaxy shear and correct it from the PSF.

3) Correct the shear from intrinsic ellipticities

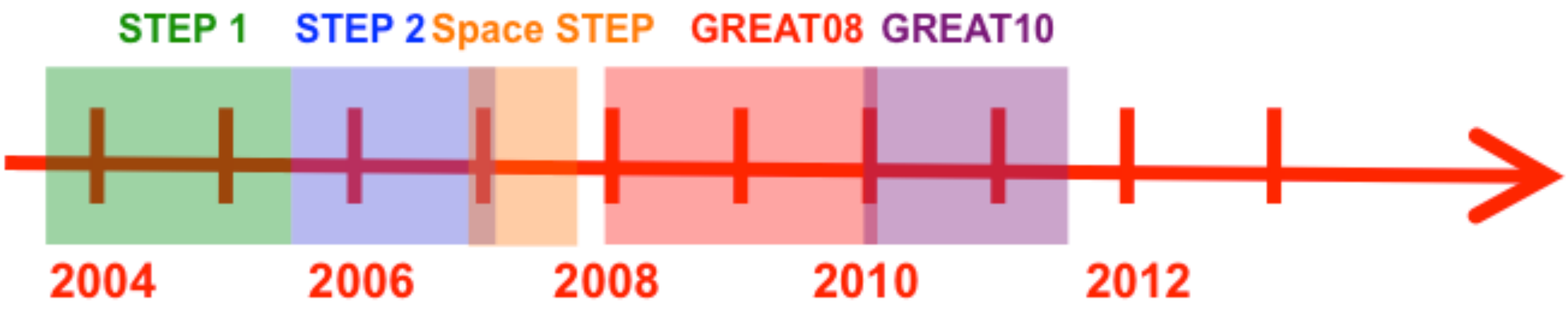
+ noise and **missing data**!!!



Missing data



Shape measurement techniques: chronology of challenges

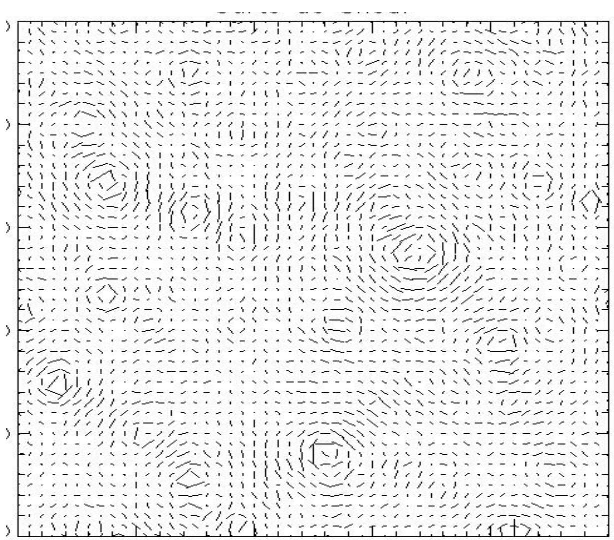


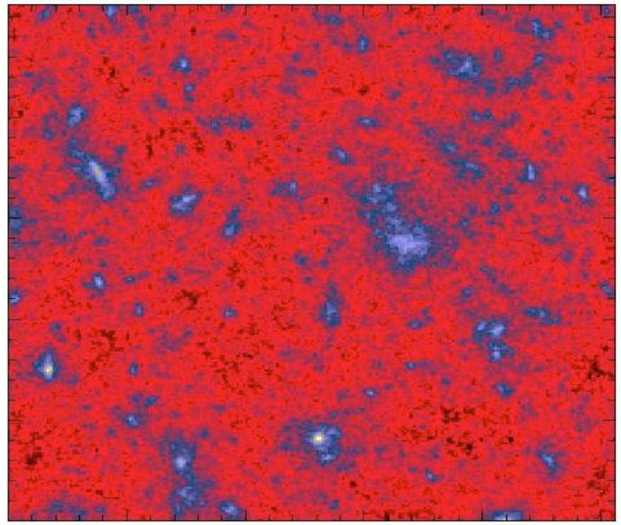
~3 more challenges should be organised before 2020

In GREAT10, there were 3 sub-challenges:

1. the main (galaxy) challenge: to measure the ellipticity of galaxies, assuming the PSF is known
2. a star challenge: to estimate the PSF and interpolate it at the position of galaxies
3. a 'light' challenge (named 'kaggle') to attract more people

Shear Field and Mass Map





$$\begin{pmatrix} \hat{E}(\boldsymbol{k}) = \hat{\kappa}(\boldsymbol{k}) \\ \hat{B}(\boldsymbol{k}) \end{pmatrix} = \underbrace{\frac{1}{|\boldsymbol{k}|^2} \begin{pmatrix} k_1^2 - k_2^2 & 2k_1 k_2 \\ 2k_1 k_2 & -k_1^2 + k_2^2 \end{pmatrix}}_{A_{\boldsymbol{\kappa}}} \begin{pmatrix} \hat{\gamma}_1(\boldsymbol{k}) \\ \hat{\gamma}_2(\boldsymbol{k}) \end{pmatrix}$$

PB 5: A simple reconstruction is very noisy

Typical Weak Lensing Pipeline

1) Take an image of the sky

2) Measure galaxy shapes and distances

3)

c) Bin the data in redshift slices

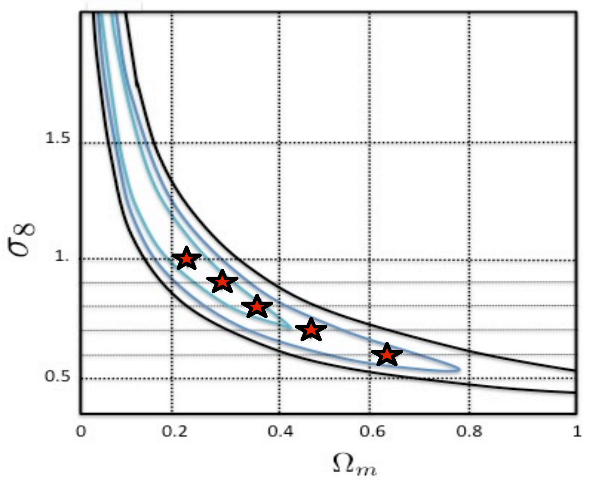
d) Correlation function tomography

a) Create a 3D shear field

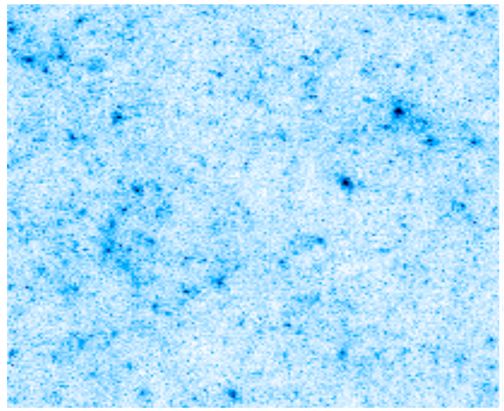
b) Reconstruct the matter distribution

4) Constrain cosmological parameters

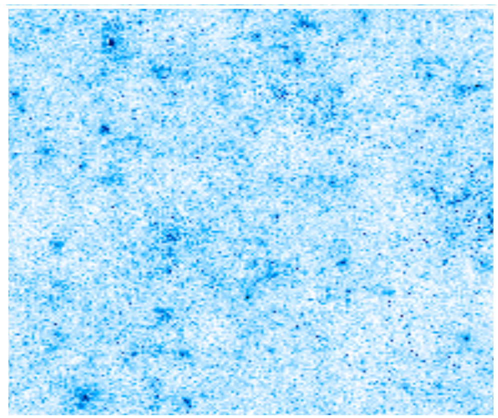
Cosmological Parameters Constraints and High Order Statistics



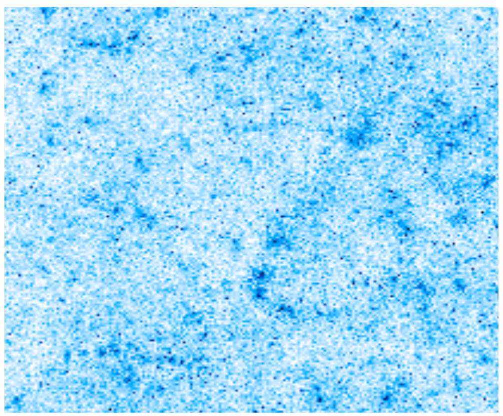
Model1 (σ_8=1, Ω_m=0.23)



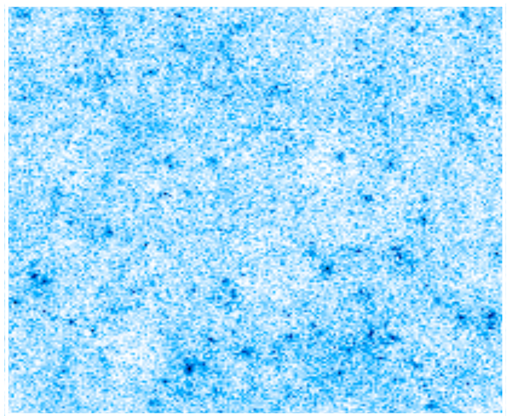
Model2 (σ_8=0.9, Ω_m=0.3)



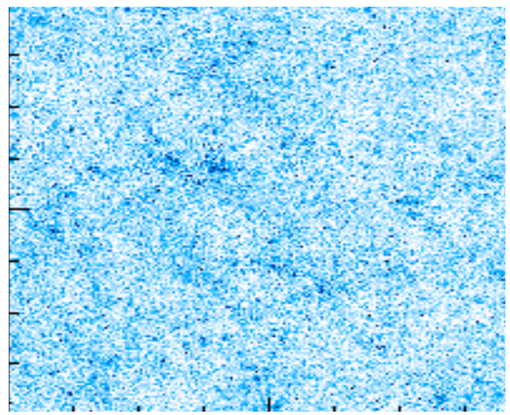
Model3 (σ_8=0.8, Ω_m=0.36)



Model4 (σ_8=0.7, Ω_m=0.47)



Model5 (σ_8=0.6, Ω_m=0.64)



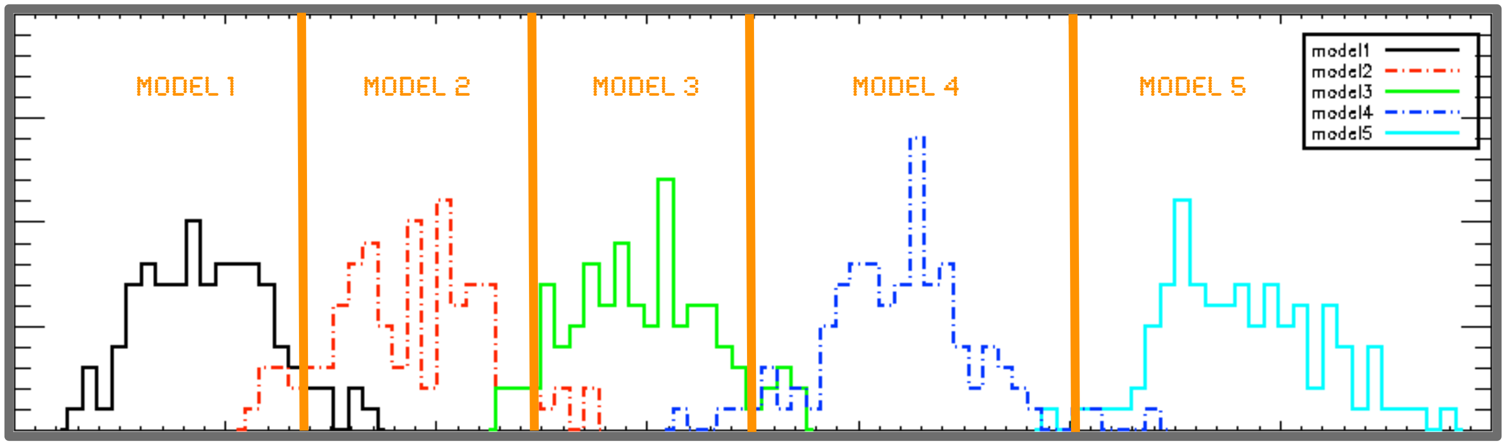
Cosmological Parameters Constraints and High Order Statistics

- Aperture mass map = wavelets, but wavelets calculation is between 10 and 1000 times faster.

- A. Leonard, S. Pires, J.-L. Starck, "Fast Calculation of the Weak Lensing Aperture Mass Statistic", MNRAS, 423, pp 3405-3412, 2012.

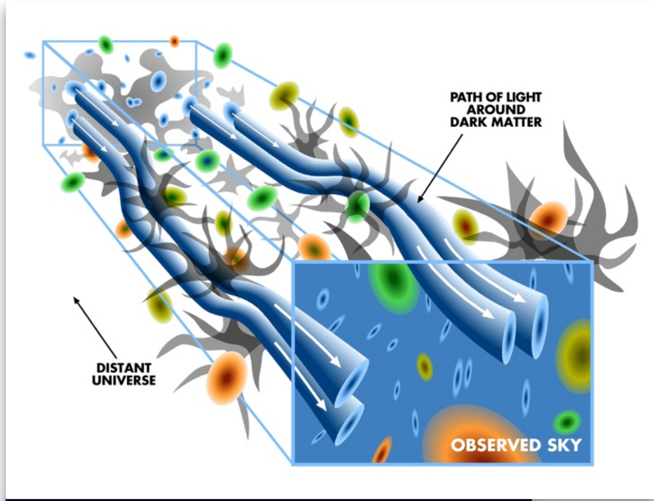
- Wavelet Denoising + Wavelet Peak Counting is the most efficient statistical tool to discriminate Cosmological Models.

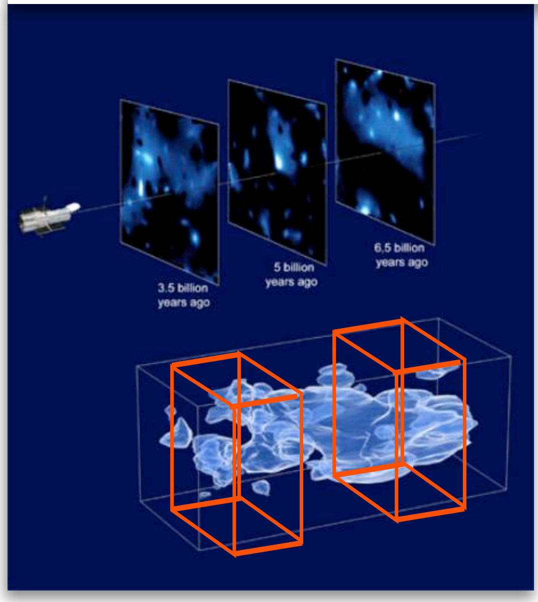
- S. Pires, A. Leonard, J.-L. Starck, "Cosmological Parameters Constraint from Weak Lensing Data", MNRAS, 423, pp 983-992, 2012.

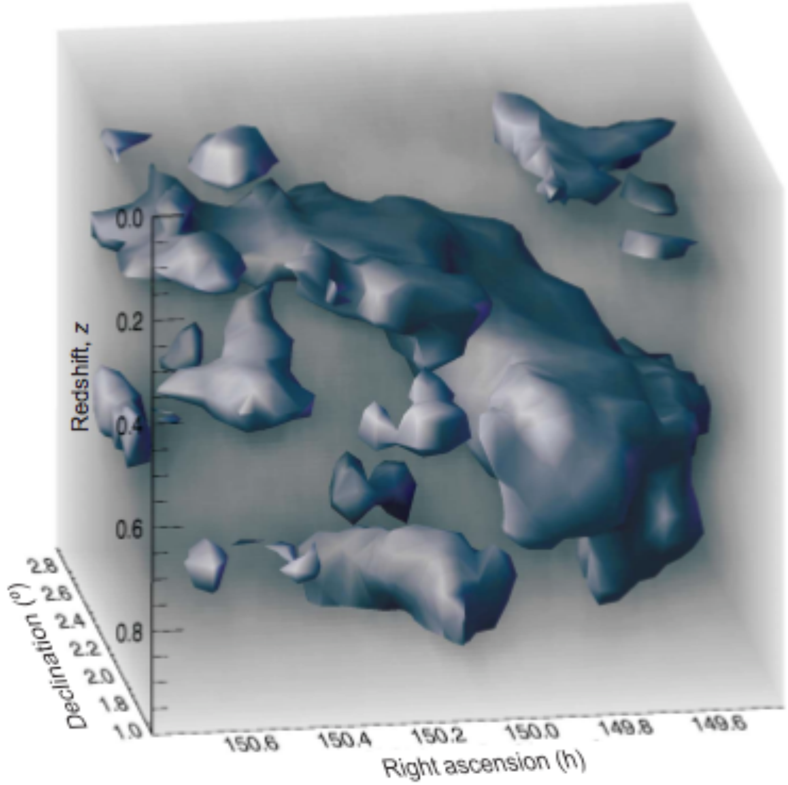


WAVELET PEAK COUNTING ON MRLENS FILTRED MAPS (AT SCALE OF ABOUT 1 ARCMIN)

Pseudo-3D Weak Lensing

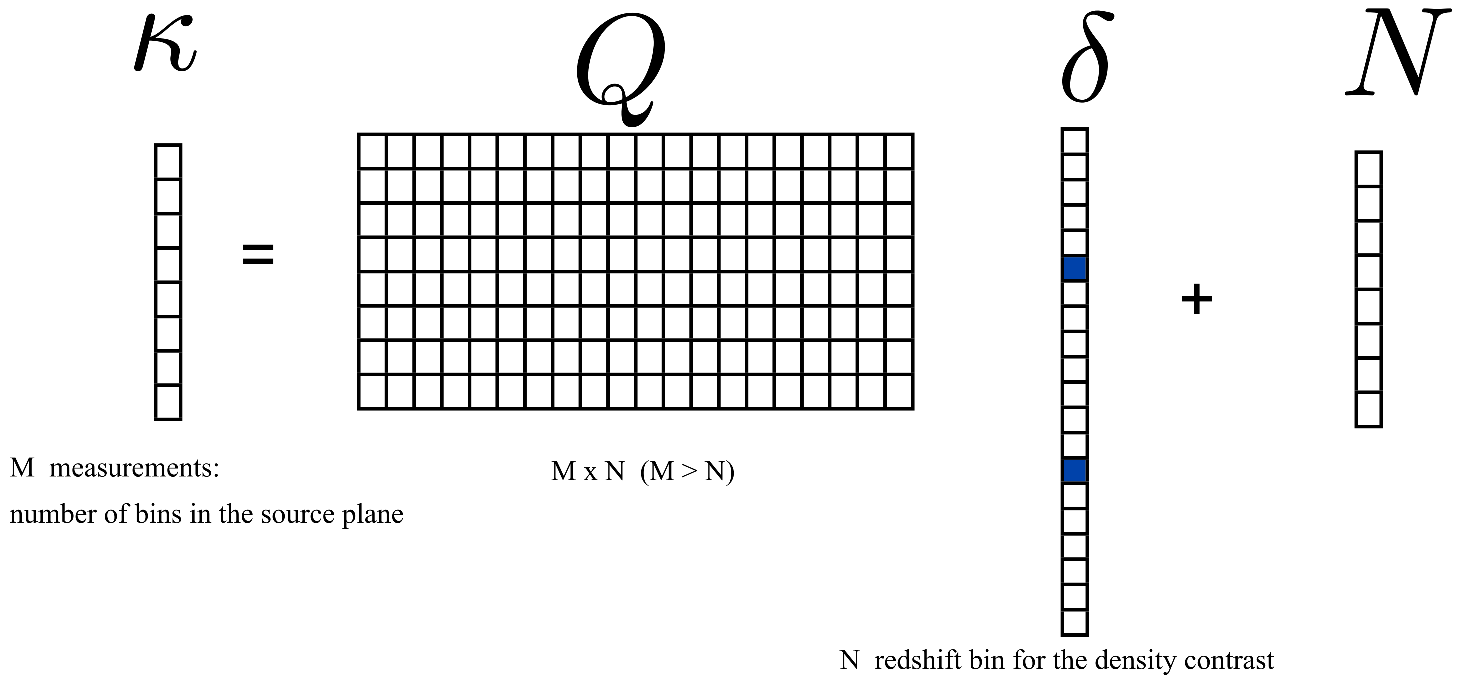






R. Massey et al, Maps of the Universe's Dark matter scaffolding,, Nature, Vol. 445, pp. 286-290, 2007

Tomographic 3D Weak Lensing



δ is sparse.

Q spreads out the information in δ along κ bins.

PB6: More unkown than measurements

3D Weak Lensing

The convergence κ, as seen in sources of a given redshift bin, is the linear transformation of the matter density contrast, δ, along the line-of-sight (Simon et al 2009):

$$\kappa = Q\delta + N$$

with $\delta(r) \equiv \rho(r)/\overline{\rho} - 1$

$$Q_{i\ell} = \frac{3H_0^2\Omega_M}{2c^2}\int_{w_\ell}^{w_{\ell+1}} dw \frac{\overline{W}^{(i)}(w) f_K(w)}{a(w)} \,, \quad \overline{W}^{(i)}(w) = \int_0^{w^{(i)}} dw' \frac{f_K(w-w')}{f_K(w')}\left(p(z)\frac{dz}{dw}\right)_{z=z(w')}$$

where H_0 is the hubble parameter, Ω_M is the matter density parameter, c is the speed of light, $a(w)$ is the scale parameter evaluated at comoving distance w, and

$$f_K(w) = \begin{cases} K^{-1/2}\sin(K^{1/2}w), & K > 0 \\ w, & K = 0 \\ (-K)^{-1/2}\sinh([-K]^{1/2}w) & K < 0 \end{cases} ,$$

gives the comoving angular diameter distance as a function of the comoving distance and the curvature, K, of the Universe.

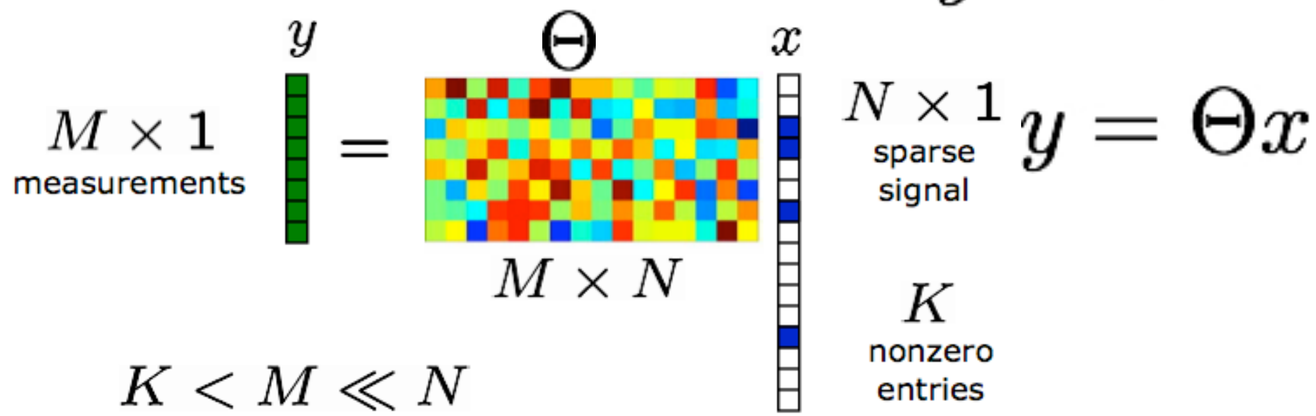
Compressed Sensing

* E. Candès and T. Tao, "Near Optimal Signal Recovery From Random Projections: Universal Encoding Strategies? ", IEEE Trans. on Information Theory, 52, pp 5406-5425, 2006.
* D. Donoho, "Compressed Sensing", IEEE Trans. on Information Theory, 52(4), pp. 1289-1306, April 2006.
* E. Candès, J. Romberg and T. Tao, "Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information", IEEE Trans. on Information Theory, 52(2) pp. 489 - 509, Feb. 2006.

A non linear sampling theorem

"Signals with exactly K components different from zero can be recovered perfectly from ~ K log N incoherent measurements"

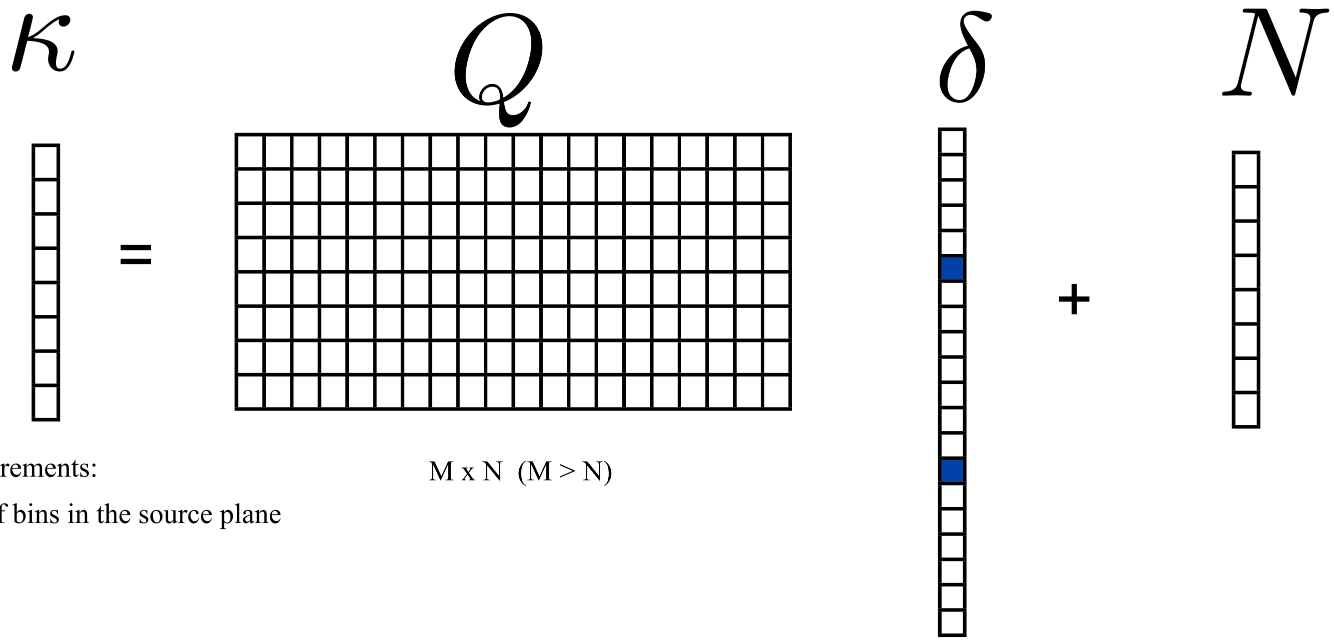
Replace samples with *few linear projections* $y = \Theta x$



Reconstruction via non linear processing: $$\min_x \|x\|_1 \quad \text{s.t.} \quad y = \Theta x$$

⇒Application: Compression, tomography, ill posed inverse problem.

3D Weak Lensing



M measurements: number of bins in the source plane

M x N (M > N)

N redshift bin for the density contrast

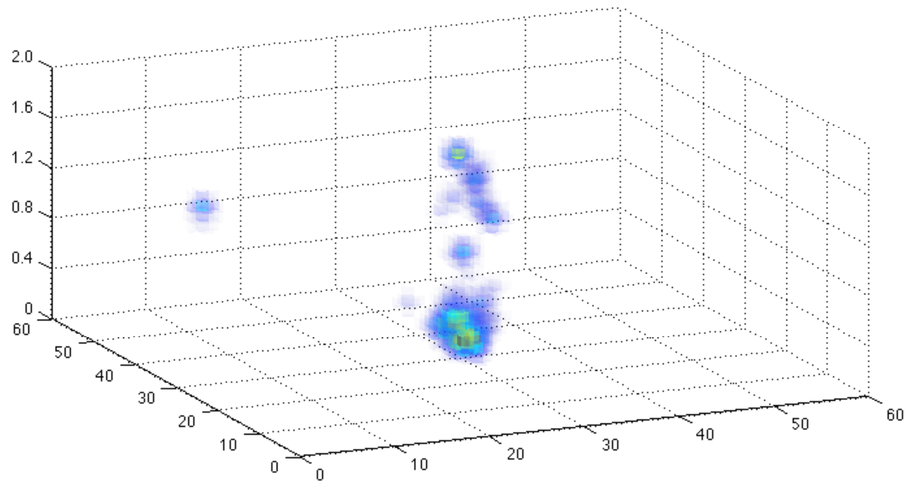
δ is sparse.

Q spreads out the information in δ along κ bins.

3D Weak Lensing

$$\min_{\delta} \parallel \delta \parallel_1 \quad s.t. \ \frac{1}{2} \parallel \kappa - Q\delta \parallel^2_{\mathbf{\Sigma}^{-1}} \leq \epsilon$$

A. Leonard, F.-X. Dupe, J.-L. Starck, "A compressed sensing approach to 3D weak lensing", Astronomy and Astrophysics, arXiv:1111.6478, A&A, 539, A85, 2012.



Reconstructions of two clusters along the line of sight, located at a redshift 0.2 and 1.0 (data binned into Nsp = 20 redshift bins, but aim to reconstruct onto Nlp = 25 redshift bins).

Conclusions

Euclid will provide:
– tight constraints over the broadest range of DE, MG models ever explored.

- Weak Lensing directly measures the mass (as opposed to light)
- But require tight control of systematic.
- Algorithms need clearly to be improved in order to meet Euclid scientific requirements.

  * Psf measurements
  * Shear on individual galaxies
  * Lensing statistics.
  * 2D convergence map and 3D density contrast map.

- Recent developments in Math&Statististics (sparsity concept, compressed sensing, proximal optimization theory, etc) will be extremely useful to optimize Euclid Science.

http://www.euclid-ec.org
http://www.cosmostat.org